Individual Competition
Aug. 23, 2017

English version

Problem I–1

Determine all functions $f\colon \mathbb{R} \to \mathbb{R}$ satisfying

$$f(x^2 + f(x)f(y)) = xf(x + y)$$

for all real numbers x and y.

Problem I–2

Let $n \geqslant 3$ be an integer. A labelling of the n vertices, the n sides and the interior of a regular n-gon by $2n + 1$ distinct integers is called *memorable* if the following conditions hold:

(a) Each side has a label that is the arithmetic mean of the labels of its endpoints.

(b) The interior of the n-gon has a label that is the arithmetic mean of the labels of all the vertices.

Determine all integers $n \geqslant 3$ for which there exists a memorable labelling of a regular n-gon consisting of $2n + 1$ consecutive integers.

Problem I–3

Let $ABCDE$ be a convex pentagon. Let P be the intersection of the lines CE and BD. Assume that $\angle PAD = \angle ACB$ and $\angle CAP = \angle EDA$. Prove that the circumcentres of the triangles ABC and ADE are collinear with P.

Problem I–4

Determine the smallest possible value of

$$|2^m - 181^n|,$$

where m and n are positive integers.

Time: 5 hours
Time for questions: 60 min
Each problem is worth 8 points.
The order of the problems does not depend on their difficulty.